1. This proof is by induction on .

Basis step: Note that , so . Thus the claim holds true for .

Induction Hypothesis: Assume the claim is true for some natural number . That is, . Add to each side to get . Note that , so . Which means . Thus .

Therefore, by induction, the statement is true for all natural numbers.

1. This proof is by induction on .

Basis Step: Note that . So, , which is odd, and , which is also odd. is even because an odd plus an odd is even. Thus the statement is true for .

Induction Hypothesis: Assume the claim is true for some positive integer . That is, is even. By the definition of the Fibonacci sequence . By expanding the RHS we get . Since we know is even and is even because it is a multiple of 2, then is even because an even plus an even is even.

Therefore, by induction, the statement is true for all positive integers.

1. Let A includes those that are equivalent to it and are four-digit natural numbers. So, the set can be notated by .
2. Suppose that and are sets with A ⊆ B. SFTOC that . Let . By definition of the set difference, and . By the definition of a subset, since and , then . Thus we have a contradiction⇒⇐. Therefore .
3. 1. There are different ways to fill the positions of the field because there are 11 different positions and there are 23 players to choose from.
   2. There are different ways to choose the forwards. Once that choice is made, there are different ways to choose the midfielders. After both choices are made, there are different ways to choose the fullbacks. Finally, after all 3 choices are made, there are different ways to choose the goalie. Thus there are different ways to choose the 4 forwards, 3 midfielders, 3 fullbacks, and 1 goalie respectively.
   3. There are Different ways to choose the players to play on the field.
4. This proof is by induction on .

Basis step: Note that because and , so the claim holds true for .

Induction Hypothesis: Assume that the claim is true for some positive integer . That is, . Add to both sides to get . Looking at the RHS, .

Therefore, by induction, the statement is true for all positive integers.